

Chirped pulse excitation of two-atom Rydberg states

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We analyze excitation of two ground state atoms to a double Rydberg state by a two-photon chirped optical pulse in the regime of adiabatic rapid passage. For intermediate Rydberg-Rydberg interaction strengths, relevant for atoms separated by $\sim 10 \mu\text{m}$, adiabatic excitation can be achieved at experimentally feasible Rabi frequencies and chirp rates of the pulses, resulting in high transfer efficiencies. We also study the adiabatic transfer between ground and Rydberg states as a means to realize a controlled phase gate between atomic qubits.

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I. INTRODUCTION

Neutral atoms in ground hyperfine states represent a clean, well-controlled system for quantum information processing providing long time qubit storage, easy initialization, readout, manipulation with optical and magnetic fields, and scalability [1]. Two-qubit gates with atoms can be realized by transferring them into high-lying Rydberg states, in which the atoms can interact via strong and long-distance van der Waals and dipole-dipole interactions [2]. In the seminal proposal [3] of a controlled phase gate for atomic qubits based on interaction in Rydberg states two realizations have been discussed. The first one works for close atoms in a regime when the interaction strength V_{int} exceeds the Rabi frequency Ω of excitation pulses coupling qubit states to the Rydberg ones: $V_{\text{int}} > \Omega$. In this regime of a Rydberg blockade the strong interaction prevents excitation of a second atom if the first atom has been excited. The blockade has been experimentally demonstrated for two atoms in separate dipole traps [4], followed by realization of a CNOT gate [5] and entanglement [6]. A second approach to the controlled phase gate, discussed in [3], applies to smaller interaction strengths $V_{\text{int}} \lesssim \Omega$, for which the blockade cannot work. The gate then can be implemented by conditionally exciting both atoms to the Rydberg state, letting them interact to accumulate a π phase shift, and de-exciting back to their original qubit states. This allows to realize the gate between atoms separated by several sites in an optical lattice architecture and between atoms in distant individual microtraps, making the systems scalable. The second approach has not yet been realized experimentally and is currently a subject of active theoretical investigation [7–9]. Interactions in Rydberg states can also find applications in quantum simulation [10, 11], quantum repeaters [12], and in the realization of efficient and non-local nonlinearities [13], down to a single photon level [14].

Coherent control techniques such as STIRAP [15] and adiabatic rapid passage (ARP) [16] can provide robust excitation to the two-atom Rydberg state. Transfer of two atoms to the double Rydberg state by STIRAP was

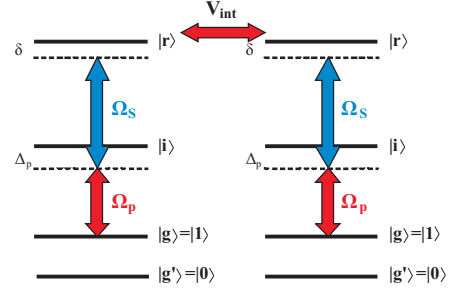


FIG. 1: Two ladder-type atomic systems interact with chirped pump (Rabi frequency Ω_p) and Stokes fields (Rabi frequency Ω_s) at $|g\rangle - |i\rangle$ and $|i\rangle - |r\rangle$ transitions, respectively. The one-photon and two-photon detunings are $\Delta_p = \omega_{ig} - \omega_p$ and $\delta = \omega_{rg} - \omega_p - \omega_s$. The atoms also experience a van der Waals or dipole-dipole interaction in the double Rydberg state $|rr\rangle$. Two long-lived ground states $|g\rangle$ and $|g'\rangle$ are used to encode qubit states $|1\rangle$ and $|0\rangle$.

studied in [7], where it was shown that in a system of three-level atoms, typically used in experiments, having ground $|g\rangle$, intermediate $|i\rangle$ and Rydberg states $|r\rangle$ (shown in Fig.1), the only dark state does not connect the two-atom ground $|gg\rangle$ and Rydberg $|rr\rangle$ states. Application of pump and Stokes pulses, resonant with the $|g\rangle - |i\rangle$ and $|i\rangle - |r\rangle$ transitions, respectively, transfers the system from $|gg\rangle$ to an entangled $(|ii\rangle - |gg\rangle)/\sqrt{2}$ state, not containing $|rr\rangle$. In a later work [9] it was found that a non-zero one-photon detuning $\Delta_p \sim V_{\text{int}}$ in the STIRAP scheme produces a dressed state directly connecting $|gg\rangle$ to $|rr\rangle$, but the transfer efficiency to that state was not optimal because of population loss from a fast decaying intermediate state. STIRAP-like excitation to the $|rr\rangle$ state with larger detunings $\Delta_p \sim \text{hundreds MHz}$ can also be realized using optimal control techniques by shaping the pulses such that the population of the intermediate state is minimized [17].

Adiabatic rapid passage with chirped optical pulses is well-known for providing efficient population transfer between quantum states, and it will be studied in this work as a means to achieve robust excitation to double Rydberg states. In the two-photon ARP excitation scheme of

Fig.1 a large one-photon detuning from the fast-decaying intermediate state can be used, allowing to minimize its population and obtain high transfer efficiencies. In section II we show that the two-atom system can be excited to the double Rydberg state along one of the dressed states, directly connecting $|gg\rangle$ to $|rr\rangle$. In section III we numerically calculate the transfer efficiency for Rb atoms taking into account decays from the intermediate and Rydberg states and analyze its dependence on Rabi frequencies and chirp rates of the pulses. Finally, in section IV we calculate the fidelity of the controlled phase gate which can be realized by conditionally transferring the ground state qubits to the $|rr\rangle$ state and back such that the qubit state $|11\rangle$ accumulates the π phase shift, and conclude in Section V.

II. DRESSED STATES FOR TWO THREE-LEVEL ATOMS INTERACTING WITH A TWO-PHOTON CHIRPED FIELD

We consider two three-level atoms with internal states $|g\rangle$, $|i\rangle$ and $|r\rangle$, shown in Fig.1. Each atom interacts with two chirped laser pulses, pump and Stokes, on the transitions $|g\rangle - |i\rangle$ and $|i\rangle - |r\rangle$, respectively. In the Rydberg states $|r\rangle$ they additionally interact with each other via dipole-dipole or van der Waals interactions. The system can be described by the Schrodinger equation assuming that all the interactions are much faster compared to decays of intermediate and Rydberg states. For simplicity of the analysis one can use "molecular" states: $|gg\rangle$, $|\pm\rangle_{ig} = (|ig\rangle \pm |gi\rangle)/\sqrt{2}$, $|ii\rangle$, $|\pm\rangle_{rg} = (|rg\rangle \pm |gr\rangle)/\sqrt{2}$, $|\pm\rangle_{ri} = (|ri\rangle \pm |ir\rangle)/\sqrt{2}$, $|rr\rangle$. Expanded in these states the two atom wave function has a form $|\Psi\rangle = c_{gg}|gg\rangle + \sum_{\sigma=\pm} c_{\sigma,ig}|\sigma\rangle_{ig} + c_{ii}|ii\rangle + \sum_{\sigma=\pm} c_{\sigma,rg}|\sigma\rangle_{rg} + \sum_{\sigma=\pm} c_{\sigma,ri}|\sigma\rangle_{ri} + c_{rr}|rr\rangle$, where the amplitudes of the "molecular" states are expressed via amplitudes of pure two atom states as follows: $c_{\pm,ig} = (c_{ig} \pm c_{gi})/\sqrt{2}$, $c_{\pm,rg} = (c_{rg} \pm c_{gr})/\sqrt{2}$, $c_{\pm,ri} = (c_{ri} \pm c_{ir})/\sqrt{2}$.

The Hamiltonian of the system in the rotating wave approximation is given by:

$$H/\hbar = \sum_{j=1,2} (\Delta_p(t) |i\rangle_j \langle i| + \delta(t) |r\rangle_j \langle r|) + V_{\text{int}} |rr\rangle \langle rr| - \sum_{j=1,2} \left(\vec{\mu}_{ig} \vec{E}_p(t)/2\hbar |i\rangle_j \langle g| + \vec{\mu}_{ri} \vec{E}_S(t)/2\hbar |r\rangle_j \langle i| + \text{H.c.} \right), \quad (1)$$

where $\Delta_p(t) = \omega_{ig} - \omega_p(t)$, $\delta(t) = \omega_{rg} - \omega_p(t) - \omega_S(t)$ are the one and two-photon detunings of the pump and Stokes field frequencies ω_p and ω_S from the atomic transition frequencies ω_{ig} and ω_{rg} , $\vec{\mu}_{ig}$ and $\vec{\mu}_{ri}$ are the dipole moments of the corresponding transitions and $\vec{E}_{p,S}$ are the amplitudes of the pump and Stokes fields. The Schrodinger equations for the "molecular" state ampli-

tudes are then given by:

$$\begin{aligned} i \frac{dc_{gg}}{dt} &= -\sqrt{2}\Omega_p c_{+,ig}, \\ i \frac{dc_{+,ig}}{dt} &= \Delta_p c_{+,ig} - \sqrt{2}\Omega_p c_{gg} - \sqrt{2}\Omega_p c_{ii} - \Omega_S c_{+,rg}, \\ i \frac{dc_{-,ig}}{dt} &= \Delta_p c_{-,ig} - \Omega_S c_{-,rg}, \\ i \frac{dc_{ii}}{dt} &= 2\Delta_p c_{ii} - \sqrt{2}\Omega_p c_{+,ig} - \sqrt{2}\Omega_S c_{+,ri}, \\ i \frac{dc_{+,rg}}{dt} &= \delta c_{+,rg} - \Omega_S c_{+,ig} - \Omega_p c_{+,ri}, \\ i \frac{dc_{-,rg}}{dt} &= \delta c_{-,rg} - \Omega_S c_{-,ig} - \Omega_p c_{-,ri}, \\ i \frac{dc_{+,ri}}{dt} &= (\delta + \Delta_p) c_{+,ri} - \sqrt{2}\Omega_S c_{ii} - \sqrt{2}\Omega_S c_{rr} - \Omega_p c_{+,rg}, \\ i \frac{dc_{-,ri}}{dt} &= (\delta + \Delta_p) c_{-,ri} - \Omega_p c_{-,rg}, \\ i \frac{dc_{rr}}{dt} &= (2\delta + V_{\text{int}}) c_{rr} - \sqrt{2}\Omega_S c_{+,ri}, \end{aligned} \quad (2)$$

where $\Omega_p = \vec{\mu}_{ig} \vec{E}_p/2\hbar$, $\Omega_S = \vec{\mu}_{ri} \vec{E}_S/2\hbar$ are the Rabi frequencies of the pump and Stokes fields. First, one can notice from Eqs.(2) that the $|+\rangle$ and $|-\rangle$ states decouple, i.e. laser fields connect only states within these subsystems. Second, the $|gg\rangle$ state is laser coupled only to $|+\rangle$ states such that if initially the atoms are in the $|gg\rangle$ state, the $|-\rangle$ subsystem is never populated, and, therefore, can be discarded in the present model which neglects decays. Next, we assume that the one-photon detuning is large $|\Delta_p| \gg \delta$, Ω_p , Ω_S such that the amplitudes $c_{+,ig}$, $c_{+,ii}$ and $c_{+,ri}$ can be replaced by their steady-state solutions as follows:

$$\begin{aligned} c_{+,ig} &\approx \frac{\sqrt{2}\Omega_p c_{gg} + \Omega_S c_{+,rg} + \sqrt{2}\Omega_p c_{ii}}{\Delta_p}, \\ c_{+,ri} &\approx \frac{\sqrt{2}\Omega_S c_{ii} + \Omega_p c_{+,rg} + \sqrt{2}\Omega_S c_{rr}}{\Delta_p}, \\ c_{ii} &\approx \frac{\sqrt{2}\Omega_p c_{+,ig} + \sqrt{2}\Omega_S c_{+,ri}}{2\Delta_p}. \end{aligned}$$

Expressing further the $c_{+,ig}$ and $c_{+,ri}$ in terms of c_{gg} , $c_{+,rg}$ and c_{rr} we eliminate the intermediate state $|i\rangle$ and reduce the system Eqs.(2) to three equations for states $|gg\rangle$, $|+\rangle_{rg}$ and $|rr\rangle$:

$$\begin{aligned} i \frac{dc_{gg}}{dt} &= -\frac{2\Omega_p^2}{\Delta_p} c_{gg} - \sqrt{2}\Omega c_{+,rg} - 2\frac{\Omega^2}{\Delta_p} c_{rr}, \\ i \frac{dc_{+,rg}}{dt} &= \left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} \right) c_{+,rg} - \sqrt{2}\Omega c_{gg} - \sqrt{2}\Omega c_{rr}, \\ i \frac{dc_{rr}}{dt} &= \left(\delta + V_{\text{int}} - \frac{2\Omega_S^2}{\Delta_p} \right) c_{rr} - \sqrt{2}\Omega c_{+,rg} - \frac{2\Omega^2}{\Delta_p} c_{gg}, \end{aligned}$$

where $\Omega = \Omega_p \Omega_S / \Delta_p$ is the two-photon Rabi frequency. One can now obtain the dressed states of the two atom

system and their energies ϵ from the energy equation:

$$-\left(\epsilon + \frac{2\Omega_p^2}{\Delta_p}\right)\left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} - \epsilon\right)\left(2\delta - \frac{2\Omega_S^2}{\Delta_p} - \epsilon + V_{\text{int}}\right) + 2\Omega^2\left(2\epsilon + 2\frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} - \frac{4\Omega^2}{\Delta_p} - 2\delta - V_{\text{int}}\right) = 0. \quad (3)$$

Neglecting the Ω^2/Δ_p term and setting $V_{\text{int}} = 0$ allows one to obtain analytical expressions for the dressed states and their energies in the absence of the Rydberg-Rydberg interaction:

$$\begin{aligned} \epsilon_1 &= \delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}, \\ \epsilon_{2,3} &= \delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} \pm \sqrt{\left(\delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right)^2 + 4\delta\frac{\Omega_p^2}{\Delta_p}} = \\ &= \delta - \frac{\Omega_p^2 + \Omega_S^2}{\Delta_p} \pm \sqrt{\delta^2 + 2\delta\frac{\Omega_p^2 - \Omega_S^2}{\Delta_p} + \left(\frac{\Omega_p^2 + \Omega_S^2}{\Delta_p}\right)^2}, \end{aligned} \quad (4)$$

where $\epsilon_{2,3}$ is related to the \pm signs, respectively.

The corresponding dressed states are the following:

$$\begin{aligned} |\Psi_1\rangle &= \frac{1}{\sqrt{\left(\delta + \frac{\Omega_p^2 - \Omega_S^2}{\Delta_p}\right)^2 + 4\Omega^2}} \left(\sqrt{2}\Omega |gg\rangle - \left(\delta + \frac{\Omega_p^2 - \Omega_S^2}{\Delta_p} \right) |+\rangle_{rg} - \sqrt{2}\Omega |rr\rangle \right), \\ |\Psi_{2,3}\rangle &= \frac{1}{2\sqrt{\left(\delta + \frac{\Omega_p^2 - \Omega_S^2}{\Delta_p}\right)^2 + 4\Omega^2}} \left(\left(\epsilon_{2,3} + \frac{2\Omega_p^2}{\Delta_p} \right) |gg\rangle - 2\sqrt{2}\Omega |+\rangle_{rg} - \left(\epsilon_{2,3} + \frac{2\Omega_p^2}{\Delta_p} \right) |rr\rangle \right). \end{aligned} \quad (5)$$

We can analyze the dressed states in the limits of a large two-photon detuning δ : $|\delta| \gg \Omega_p^2/\Delta_p, \Omega_S^2/\Delta_p$:

$$\begin{aligned} 1) \quad \delta > 0: \\ |\Psi_1\rangle &\approx -|+\rangle_{rg}, \quad |\Psi_2\rangle \approx -|rr\rangle, \quad |\Psi_3\rangle \approx |gg\rangle, \\ 2) \quad \delta < 0: \\ |\Psi_1\rangle &\approx |+\rangle_{rg}, \quad |\Psi_2\rangle \approx -|gg\rangle, \quad |\Psi_3\rangle \approx |rr\rangle, \end{aligned} \quad (6)$$

which shows that $|\Psi_3\rangle \approx |gg\rangle$ for large $\delta > 0$ and $|\Psi_3\rangle \approx |rr\rangle$ for large $\delta < 0$. As a result, one can transfer the two atoms from the $|gg\rangle$ state to the $|rr\rangle$ state using a negative chirp $d\delta/dt < 0$ and back from the $|rr\rangle$ to the $|gg\rangle$ state using a positive chirp $d\delta/dt > 0$. The same can be done using the $|\Psi_2\rangle$ state and a positive chirp to realize the $|gg\rangle \rightarrow |rr\rangle$ transfer, and a negative chirp to bring the system back into $|gg\rangle$.

Above it was assumed that the interaction in the Rydberg states is zero, which allowed us to obtain analytical

expressions for dressed states and their energies. When $V_{\text{int}} \neq 0$ the energies and dressed states can be calculated only numerically. We are interested in the case $V_{\text{int}} \sim \Omega$, when the dipole blockade is not working, and investigate how two atoms can be transferred to the double Rydberg state in this regime. The energies and the amplitudes of the $|gg\rangle$, $|+\rangle_{rg}$ and $|rr\rangle$ components of the $|\Psi_2\rangle$ and $|\Psi_3\rangle$ states in the case $V_{\text{int}} = 0.5\Omega$ are shown in Fig.2a,b and c, respectively. One can see that for a large positive δ $|\Psi_3\rangle \approx |gg\rangle$ and for a large negative δ $|\Psi_3\rangle \approx |rr\rangle$ (for a large negative δ $|\Psi_2\rangle \approx |gg\rangle$ and for a large positive δ $|\Psi_2\rangle \approx |rr\rangle$), as expected from Eq.(6). Fig.2 shows, therefore, that for intermediate interaction strengths the system can still be transferred from $|gg\rangle$ to $|rr\rangle$ if it adiabatically follows either $|\Psi_3\rangle$ for a negative chirp rate or $|\Psi_2\rangle$ for a positive one.

III. EFFICIENCY OF EXCITATION TO DOUBLE RYDBERG STATE BY ARP

In this section we numerically analyze the efficiency of two-photon excitation from $|gg\rangle$ to a double Rydberg state $|rr\rangle$ using adiabatic rapid passage. We consider two three-level atoms interacting with pump and Stokes optical pulses and with each other via vdW or dipole-dipole interaction according to the Hamiltonian (1). We also take into account radiative decays from the intermediate $|i\rangle$ and Rydberg $|r\rangle$ states and describe the system using a density matrix equation:

$$\frac{d\rho}{dt} = \frac{i}{\hbar} [\rho, H] + \mathcal{L}\rho, \quad (7)$$

where the Lindblad term, incorporating the decays, is as follows:

$$\begin{aligned} \mathcal{L}\rho &= \sum_{j=1,2} \frac{\Gamma_i}{2} \left(2\sigma_{ig}^{-j} \rho \sigma_{ig}^{+j} - \sigma_{ig}^{+j} \sigma_{ig}^{-j} \rho - \rho \sigma_{ig}^{+j} \sigma_{ig}^{-j} \right) + \\ &+ \sum_{j=1,2} \frac{\Gamma_r}{2} \left(2\sigma_{ri}^{-j} \rho \sigma_{ri}^{+j} - \sigma_{ri}^{+j} \sigma_{ri}^{-j} \rho - \rho \sigma_{ri}^{+j} \sigma_{ri}^{-j} \right), \end{aligned} \quad (8)$$

where $\sigma_{ig}^{+j} = |i\rangle_j \langle g|$, $\sigma_{ri}^{+j} = |r\rangle_j \langle i|$ and $\sigma_{ig}^{-j} = \left(\sigma_{ig}^{+j} \right)^\dagger$, $\sigma_{ri}^{-j} = \left(\sigma_{ri}^{+j} \right)^\dagger$ are the raising and lowering operators for the j^{th} atom, Γ_i and Γ_r are radiative decay rates of the intermediate and Rydberg states. The pump and Stokes pulses Rabi frequencies have a Gaussian form $\Omega_{p,S}(t) = \Omega_{0p,0S} \exp(-(t - t_c)^2/2\tau_{p,S}^2)$ and detunings are $\Delta_p(t) = \omega_{ig} - \omega_p(t) = \Delta_{0p} + \alpha(t - t_c)$, $\Delta_S(t) = \omega_{ri} - \omega_S(t) = \Delta_{0S} + \alpha(t - t_c)$, where α is the linear chirp rate of the pulses, assumed equal for both.

The population ρ_{rr} of the double Rydberg state, i.e. the excitation efficiency, is shown in Fig.3 for a range of two-photon Rabi frequencies and chirp rates. In calculations parameters of ^{87}Rb atoms were used with $|i\rangle = 5P_{3/2}$ with decay rate $\Gamma_i = 6$ MHz and $|r\rangle = 80S$ with $\Gamma_r = 485$

Hz, which included the decay due to spontaneous emission (~ 300 Hz) and due to interaction with black-body radiation at $T = 300$ K (~ 185 Hz). One can see that the efficiency reaches $\sim 97\%$ for sufficiently high two-photon Rabi frequencies and chirp rates, providing adiabatic interaction between atoms and laser pulses. Adiabaticity requires that $|d\delta/dt| \ll \Omega^2$ and $|d\delta/dt|\tau_{p,S}^2 \gg 1$ [19], where $d\delta/dt = 2\alpha$ is the two-photon chirp rate, as well as equal Rabi frequencies $\Omega_p = \Omega_S$, pulse durations $\tau_p = \tau_S$ and chirp rates of the pump and Stokes pulses. From Fig.3a one can see that the efficiency becomes high ($\sim 90\%$) for $\alpha \geq 300$ MHz/ μ s and $\Omega \geq 30$ MHz because for these parameters (using pulse durations $\tau_p = \tau_S \sim 0.1$ μ s) the adiabaticity conditions are satisfied: $\Omega^2/2\alpha \approx 4.5$ and $2\alpha\tau_{p,S}^2 \approx 38$. Fig.4 gives the time evolution of populations of the $|gg\rangle$, $|+\rangle_{rg}$ and $|rr\rangle$ states during the excitation. The parameters of the pulses providing efficient transfer are challenging but within experimental reach: currently the Rabi frequencies of the pump and Stokes pulses can be increased up to $\Omega_{0p,0S} \sim 250$ MHz [20], while the chirp rates can be as high as ~ 40 GHz/ μ s [21].

IV. CONTROLLED PHASE GATE USING ARP EXCITATION TO DOUBLE RYDBERG STATE

The interaction in Rydberg states can be used to realize a controlled phase gate C_Z , which acts on two-qubit states $|nm\rangle$ ($n, m = 0, 1$) as $|nm\rangle \rightarrow e^{i\pi nm} |nm\rangle$. The gate can be implemented using ARP excitation in the following way. In the first step four-level atoms, shown in Fig.1, interact with a chirped two-photon pulse such that the system initially in $|gg\rangle = |11\rangle$ evolves along the $|\Psi_3\rangle$ dressed state. At the end of the pulse the atoms will be transferred into $|rr\rangle$ and acquire a phase factor $\int \epsilon_3(t)dt$. At the same time the states $|gg'\rangle = |10\rangle$ and $|g'g\rangle = |01\rangle$ will evolve into $|r0\rangle$ and $|0r\rangle$, respectively, and acquire a phase factor $\int \epsilon_{\pm}(t)dt$, where $\epsilon_{\pm} = \delta/2 - (\Omega_p^2 + \Omega_S^2)/2\Delta_p \pm \sqrt{(\delta/2)^2 + \Omega^2}$ are the dressed state energies for a single atom interacting with the two-photon chirped pulse [22]. One can see from Eq.(4) that if $V_{\text{int}} = 0$ and $\Omega_p = \Omega_S$, $\epsilon_{2,3} = 2\epsilon_{\pm}$ as expected. When $V_{\text{int}} \neq 0$ we can estimate the eigenenergies using Eq.(3) in the limit of small interaction strengths assuming $V_{\text{int}} \ll |\delta|$, $\Omega_p^2/|\Delta_p|$, $\Omega_S^2/|\Delta_p|$:

$$\begin{aligned} \epsilon_{2,3} &= \delta - \frac{2\Omega_p^2}{\Delta_p} \pm \sqrt{\left(\delta + \frac{V_{\text{int}}}{2}\right)^2 + 4\Omega^2} \approx \\ &\approx \delta - \frac{2\Omega_p^2}{\Delta_p} \pm \sqrt{\delta^2 + 4\Omega^2} \pm \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} = \\ &= 2\epsilon_{\pm} \pm \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}}, \end{aligned} \quad (9)$$

where $\Omega_p = \Omega_S$ was set again. It shows that for $V_{\text{int}} \neq 0$, the phase accumulated by the state $|11\rangle$, when it is

transferred to $|rr\rangle$, differs from twice the phase of the states $|01\rangle$ and $|10\rangle$ when they are transferred to $|0r\rangle$ and $|r0\rangle$, respectively. We also assume that the state $|00\rangle$ is not interacting with the pulse, which can be realized e.g. by choosing a specific polarization of the pump field. As a result, the state $|00\rangle$ will acquire zero phase.

In the second step the system returns back into qubit subspace with useful phases. To realize it one can apply the following trick [23]: (i) the chirp in the first and the second steps has the same sign such that $\delta(T+t) = -\delta(T-t)$, where T is the time boundary between the steps, as shown in Fig.5a. The system then returns from $|rr\rangle$ to $|gg\rangle$ along $|\Psi_2\rangle$, and from $|r0\rangle$ and $|0r\rangle$ to $|10\rangle$ and $|01\rangle$ along ϵ_+ ; (ii) the one-photon detuning changes sign in the second step with respect to the first one $\Delta_p \rightarrow -\Delta_p$. Provided the pump and Stokes pulses are applied symmetrically in time $\Omega_{p,S}(T+t) = \Omega_{p,S}(T-t)$ (see Fig.5a), the conditions (i) and (ii) allow to cancel the overall phase accumulated by the $|10\rangle$ and $|01\rangle$ states: $\phi_{01} = \phi_{10} = \int_{\text{stepI}} \epsilon_{-}(t)dt + \int_{\text{stepII}} \epsilon_{+}(t)dt = 0$. At the same time, the phase accumulated by the $|11\rangle$ state $\phi_{11} = \int_{\text{stepI}} \epsilon_3(t)dt + \int_{\text{stepII}} \epsilon_2(t)dt \neq 0$. By adjusting the pulse parameters and interpulse time $\phi_{11} = \pi(2n+1)$ can be realized, which will produce the gate. For small interaction strengths one can estimate the ϕ_{11} phase using Eq.(9):

$$\begin{aligned} \phi_{11} &= \int_{T-\tau_{\text{st}}}^T \epsilon_3(t)dt + \int_T^{T+\tau_{\text{st}}} \epsilon_2(t)dt = \\ &= \int_{T-\tau_{\text{st}}}^T \left(2\epsilon_{-}(t) - \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} \right) dt + \\ &+ \int_T^{T+\tau_{\text{st}}} \left(2\epsilon_{+}(t) + \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} \right) dt = \\ &= - \int_{T-\tau_{\text{st}}}^T \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt + \int_T^{T+\tau_{\text{st}}} \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt = \\ &= 2 \int_T^{T+\tau_{\text{st}}} \frac{\delta V_{\text{int}}}{2\sqrt{\delta^2 + 4\Omega^2}} dt, \end{aligned} \quad (10)$$

where τ_{sp} is the time duration of each step, assumed equal, and $\delta(T+t) = -\delta(T-t)$ was applied in the last line. Using linearly chirped pulses with $\delta = 2\alpha(t-t_c)$ where $T < t_c < T + \tau_{\text{st}}$ and setting $\Omega = \text{const}$ for simplicity we have

$$\begin{aligned} \phi_{11} &= \frac{V_{\text{int}}}{\alpha} \sqrt{\delta^2 + 4\Omega^2} \Big|_T^{T+\tau_{\text{st}}} = \\ &= \frac{V_{\text{int}}}{\alpha} \left(\sqrt{4\alpha^2(T+\tau_{\text{st}}-t_c)^2 + 4\Omega^2} - \sqrt{4\alpha^2(T-t_c)^2 + 4\Omega^2} \right). \end{aligned}$$

In the limit $\alpha|T-t_c| \gg \Omega$ one obtains $\phi_{11} \approx 2V_{\text{int}}\tau_{\text{st}}$, showing that the duration of each step is $\tau_{\text{st}} = \pi/2V_{\text{int}}$, of the same order as the time between two STIRAP pulse sequences required to accumulate a π phase shift in [9]. For small interaction strengths the gate duration can become comparable to the Rydberg state decay time, which will reduce gate

fidelity. High fidelities are expected for intermediate strengths $V_{\text{int}} \sim \Omega$, and Fig.5b shows the fidelity $F = \langle \Psi_{\text{ideal}} | \rho(T + \tau_{\text{st}}) | \Psi_{\text{ideal}} \rangle$ numerically calculated in this regime, where the expected state of the system in the absence of errors is $|\Psi_{\text{ideal}}\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$ and $\rho(T + \tau_{\text{st}})$ is the density matrix of the system at the end of the second step taking into account population and coherence decays. An initial state of the system $|\Psi(T - \tau_{\text{st}})\rangle = \frac{1}{2}(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$ was used and the density matrix evolution was modelled by Eq.(7) with decays given by the Lindblad term (8), in which we assumed for simplicity that the population in the Rydberg state decayed to the intermediate state and in the intermediate state to $|g\rangle = |1\rangle$, i.e. there was no decay into the $|g'\rangle = |0\rangle$ state. It was also assumed that the $|0\rangle$ state was not interacting with the chirped pulse. The same parameters of ^{87}Rb were used: the radiative decay rates $\Gamma_i = 6$ MHz and $\Gamma_r = 485$ Hz, corresponding to the $5P_{3/2}$ and $80S$ states [24], respectively, and the energy splitting between the qubit states of 6.835 GHz, corresponding to the hyperfine splitting between ground state $F = 2$ and $F = 1$ sublevels. During calculations the conditions of the first and second steps were applied: the Rabi frequencies were symmetric, and chirp rate and one-photon detuning antisymmetric with respect to the step time boundary. In order to accumulate the π phase shift in the $|11\rangle$ state the time delay between two pulse sequences $2(t_c - T)$ was adjusted every time V_{int} was changed. Fig.5c shows time dependence of the dressed state energy during both steps for $V_{\text{int}} = 0.5\Omega$, with the phase ϕ_{11} given by the integral $\int_{T-\tau_{\text{st}}}^{T+\tau_{\text{st}}} \epsilon(t) dt$. Fidelities $\sim 94\%$ were obtained for $V_{\text{int}}/\Omega \sim 0.5$, limited by an incomplete conversion of $|gg\rangle$ into $|rr\rangle$ during the first step, i.e. less than 100% transfer efficiency ρ_{rr} , which resulted in a small admixture of the $|\Psi_3\rangle$ dressed state to the $|\Psi_2\rangle$ state during the second step, when the system returned from $|rr\rangle$ to $|gg\rangle$. The obtained fidelities are comparable to the ones expected in the STIRAP-based excitation scheme [9]. We checked the effect of the intermediate state decay on the fidelity, which was an important source of error in [9], and found that complete cancellation of the decay gives the fidelity increase $\sim 1\%$. The intermediate state decay is less important in our scheme due to a large one-photon detuning. Decay from the Rydberg state also does not significantly affect the gate in our case due to its short duration $\sim 2(t_c - T) \approx 300$ ns and small decay rate $\Gamma_r = 300$ Hz.

Another possible way to implement the controlled phase gate is to follow the same dressed state, e.g. the $|\Psi_3\rangle$, in both steps, which can be done if the chirp rate changes sign in the second step such that $\delta(T + t) = \delta(T - t)$. In this case at the end of the second step the state $|11\rangle$ will acquire the phase shift $\phi_{11} = \int_{T-\tau_{\text{st}}}^{T+\tau_{\text{st}}} \epsilon_3(t) dt$, and the $|01\rangle$ and $|10\rangle$ states will acquire the shift $\phi_{10} = \phi_{01} = \int_{T-\tau_{\text{st}}}^{T+\tau_{\text{st}}} \epsilon_-(t) dt$. The simplest way to realize this is to adjust pulse parameters

in such a way that $\phi_{01,10} = \pi$ and $\phi_{11} = 3\pi$ (for small V_{int} $\phi_{11} \approx 2\phi_{10,01}$, for larger V_{int} $\phi_{11} > 2\phi_{01,10}$). However, this scheme might be more challenging than the one discussed above, because two phases have to be simultaneously tuned to specific values.

The above fidelity calculations assume that the atomic motion is frozen during the gate. This assumption can be violated due to mechanical forces acting on atoms in the double Rydberg state [25]. The forces can result in excitation of higher motional states for atoms trapped in microtraps and optical lattices, resulting in undesirable entanglement between the motional and qubit states. We estimate the probability of excitation from a ground to the first excited motional state for atoms in an optical lattice. The amplitude of the first motional state after the system is deexcited from the double Rydberg state is $c_{\text{mot}} \sim \mathcal{F}\delta r(1 - \exp(-2i\omega_0(t_c - T)))/\omega_0$, where $\mathcal{F} = \partial V_{\text{int}}/\partial r = 6C_6/r^7 = 6V_{\text{int}}/r$, is the force acting between two atoms interacting via the van der Waals C_6/r^6 interaction, ω_0 is the trapping potential oscillation frequency and $\delta r \sim \sqrt{\hbar/m\omega_0}$ is the ground motional state wavefunction width, r is the distance between atoms, and $2(t_c - T)$ is the time the system spends in the double Rydberg state. Assuming $V_{\text{int}} = 5$ MHz, corresponding to $r = 9.3 \mu\text{m}$ for atoms in the $80S$ state [26], $\omega_0 = 100$ kHz, $\delta r = 35$ nm and $t_c - T \approx 150$ ns, the probability of the motional state excitation $|c_{\text{mot}}|^2 \sim 0.02$, which gives the additional error in the fidelity.

Our analysis shows that the ARP and STIRAP-type excitation [9] to the double Rydberg state, which use simple analytic pulse sequences, predict similar high controlled phase gate fidelities $\sim 94\%$ and $\sim 97\%$ in the former and the latter cases, respectively. However, these values are not good enough to allow fault-tolerant quantum computation, which requires the gate fidelity $> 99.9\%$ [27]. One of the strategies to increase the fidelity is to apply more complex coherent control techniques such as optimal control [28] and genetic [29] algorithms to shape laser pulses. Optimal control of STIRAP based blockaded controlled phase gate has been analyzed recently in [20], where it was shown that the gate error can be decreased by an order of magnitude (from 10^{-3} to 10^{-4}) if one uses optimized pulse sequences instead of analytic. Optimization of chirped pulses, first proposed in [30], is successfully used to achieve efficient population transfer between molecular states [31] and might help to improve the fidelity of the ARP based gate, which will be the subject of a future work.

V. CONCLUSION

In conclusion, we analyzed excitation of two ground state atoms to a double Rydberg state by a chirped two-photon pulse using adiabatic rapid passage. During ARP-type excitation dressed states of the coupled atoms-light system provide direct connection of the $|gg\rangle$ to the double Rydberg $|rr\rangle$ state contrary to the case of reso-

nant STIRAP [7]. Numerical analysis taking into account population and coherence decays predicts robust transfer to the $|rr\rangle$ state that can reach high efficiency $\sim 97\%$ in the case of ^{87}Rb atoms for intermediate interaction strengths $V_{\text{int}} \sim \Omega$. The high transfer efficiency is possible due to a large one-photon detuning allowed in the ARP scheme, minimizing losses from the fast-decaying intermediate state. The large one-photon detuning has to be compensated by high Rabi frequencies ~ 200 MHz of the pump and Stokes pulses to achieve adiabaticity, but they are currently within experimental reach along with required chirp rates \sim hundreds MHz/ μs . We also considered a controlled phase gate for two atomic qubits based on ARP transfer to interacting Rydberg states. Applying antisymmetric one- and two-photon detunings and symmetric Rabi frequencies during excitation and deexcitation steps one can cancel the phases of the $|01\rangle$ and $|10\rangle$ qubit states and tune the phase of the $|11\rangle$ state to π , producing the gate. Gate fidelities $\sim 94\%$ were numerically predicted at $V_{\text{int}} \sim \Omega$ for ^{87}Rb atoms,

limited by incomplete switching of the dressed states between the excitation and deexcitation steps. Our analysis shows that ARP and STIRAP-type double Rydberg state excitations using simple analytic pulse sequences are expected to achieve comparable transfer efficiencies and controlled phase gate fidelities, which are high but still insufficient for fault-tolerant quantum computation. One of the ways to increase the transfer efficiency and therefore the gate fidelity is to use more complex optimized chirped pulses.

VI. ACKNOWLEDGEMENTS

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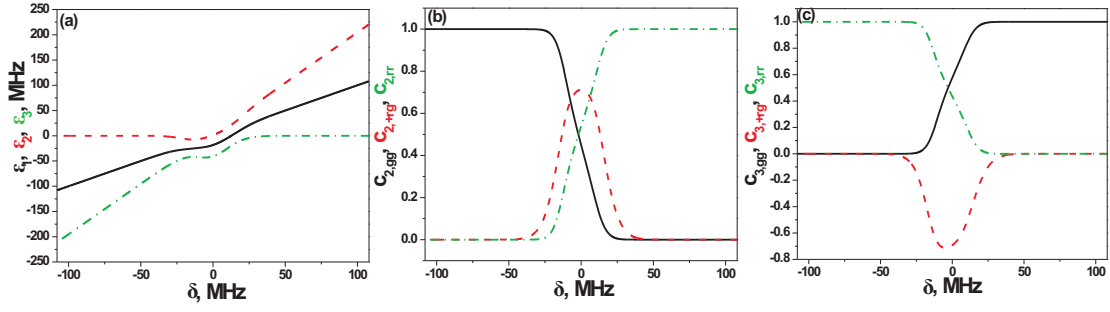


FIG. 2: (a) Energies ϵ_1 , ϵ_2 and ϵ_3 of the dressed states from Eq.(3), shown by solid black, dashed red and dash-dotted green lines, respectively; (b), (c) Components of the dressed states $|\Psi_{2,3}\rangle = c_{2,3,gg}|gg\rangle + c_{2,3,rg}|+,rg\rangle + c_{2,3,rr}|rr\rangle$, where $c_{2,3,gg}$, $c_{2,3,rg}$ and $c_{2,3,rr}$ are given by solid black, dashed red and dashed-dotted green curves, respectively. The Rabi frequencies and the detuning were parametrized as $\Omega_{p,S} = \Omega_{0p,0S} \exp(-(t - t_c)^2/2\tau_{p,S}^2)$ and $\delta = 2\alpha(t - t_c)$ with $\Omega_{0p} = \Omega_{0S} = 120$ MHz, $\tau_p = \tau_S = 75$ ns, $\alpha = 190$ MHz/ μ s, $\Delta_p = 1.5$ GHz, $V_{\text{int}} = 5$ MHz.

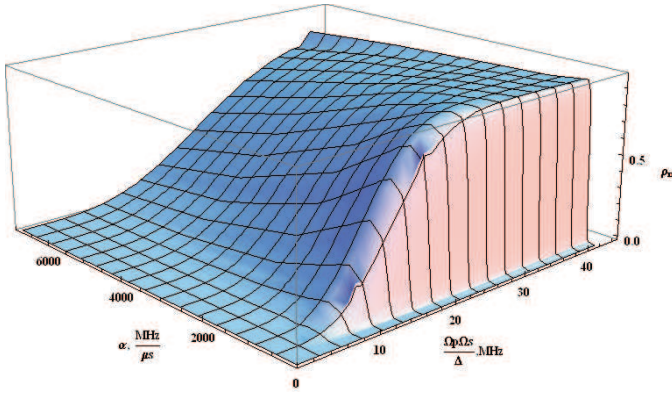


FIG. 3: Transfer efficiency to the double Rydberg state, given by the population ρ_{rr} , as a function of the two-photon Rabi frequency and chirp rate. In the calculations ^{87}Rb atoms were assumed with $|i\rangle=5P_{3/2}$ with decay rate $\Gamma_i = 6$ MHz and $|r\rangle = 80S$ with $\Gamma_r = 485$ Hz. Pump and Stokes pulses had Rabi frequencies $\Omega_{p,S} = \Omega_{0p,0S} \exp(-(t - t_c)^2/2\tau_{p,S}^2)$ with $\Omega_{0p} = \Omega_{0S} = 250$ MHz, $\tau_p = \tau_S = 100$ ns, $t_c = 20/\Gamma_i$ and detunings $\Delta_{p,S} = \Delta_{0p,0S} + \alpha(t - t_c)$ with $\alpha = 475$ MHz/ μ s. Other parameters were the following: $V_{\text{int}} = 50$ MHz, $\Delta_{0p} = 2.19$ GHz, $\Delta_{0S} = -2.268$ GHz.

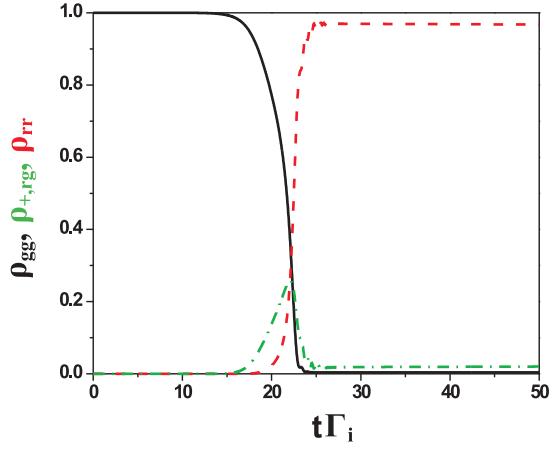


FIG. 4: Time dependence of populations of the states $|gg\rangle$ (solid black curve), $|+\rangle_{rg}$ (green dash-dotted curve) and $|rr\rangle$ (red dashed curve) during transfer. In the calculations the same parameters of ^{87}Rb atoms were assumed as in Fig.3.

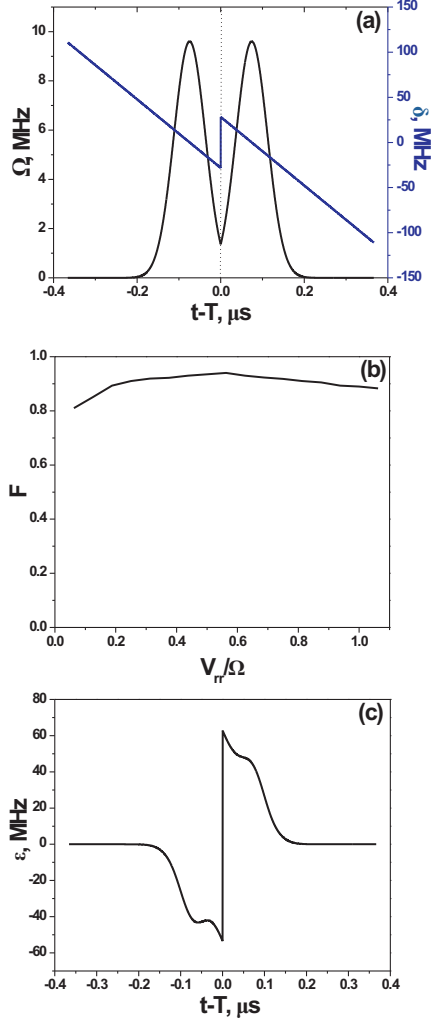


FIG. 5: (a) Time dependence of the two-photon Rabi frequency and chirp rate during the controlled phase gate for $V_{\text{int}} = 5$ MHz; (b) Fidelity of the controlled phase gate. In the calculations ^{87}Rb atoms were assumed with $|i\rangle=5P_{3/2}$ with decay rate $\Gamma_i = 6$ MHz and $|r\rangle = 80S$ with $\Gamma_r = 485$ Hz. Parameters of the pulses: $\Omega_{0p} = \Omega_{0s} = 120$ MHz, $\tau_p = \tau_s = 75$ ns, $\alpha = 190$ MHz/ μs , $\Delta_p = 1.5$ GHz; (c) Time dependence of the dressed state energy during the gate for the same interaction strength. The integral $\int_{T-\tau_{\text{st}}}^{T+\tau_{\text{st}}} \epsilon(t) dt$ gives the phase $\phi_{11} = \pi$.